

Theory of Statistics by Mark J. Schervish, Springer-Verlag GmbH & Co.
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The book under review presents a course on basic theoretical statistics comprising estimation, testing and large sample theory from both classical and Bayesian viewpoints for mathematically mature graduate students.

Chapter 1 deals with probability models including a brief review of the ideas related to their choice and analysis. Parametric models are formally introduced conforming to Bayesian outlook and appear as conditional probability measures given the parameter conceived as a random object. Bulk of this chapter is devoted to DeFinetti's representation theorem for exchangeable random variables in support of this formal idea in the case of IID parametric models and tail free processes as examples of priors for an infinite dimensional parameter.

Chapter 2 deals with sufficient statistics and gives basic results concerning sufficiency and its relationship with completeness, ancillarity and information besides discussions on the use of ancillary statistics and a general study of exponential families and information (Fisher and Kullback-Leibler). This chapter also includes derivation of conjugate priors for parametric models admitting a sufficient statistics of fixed dimension for all sample sizes, derivation and discussion of Jeffreys' prior and a generalization of DeFinetti's representation theorem to partially exchangeable random variables which uses the notion of a sufficient statistic and provides support for the Bayesian notion in the case of non IID parametric models.

Chapter 3 deals with decision theory and gives basic results on admissibility, minimaxity and complete classes besides the role of sufficiency including Rao-Blackwell theorem, Neyman-Pearson Lemma as a complete class theorem and admissibility of the sample mean in the normal case when dimension is 1 or 2 as well as the inadmissibility result demonstrated by the James-Stein estimator when the dimension is greater than 2. This chapter also presents an axiomatic decision theory which justifies the expected loss principle.

Chapter 4 deals with hypothesis testing and discusses the Neyman-Pearson theory in detail including UMP test for the MLR families and UMPU test for exponential families with critical appraisal against the decision theoretic approach and the concept of coherence and contains a proof of Bayes' character and admissibility of F -test of linear hypothesis in the univariate normal case besides the use of pure significance test, LR-test, Bayes' factor and P -values.

Chapter 5 deals with estimation and presents in the traditional style MVUE along with Lehmann-Scheffe theorem, Stein's theorem and results on lower bound for variance including Cramer-Rao and Bhattacharya bounds in the regular case and Keifer bound in the non regular case, MLE, robust estimation

including M -estimation and UMAU confidence sets besides prediction sets, tolerance sets and the use of bootstrap.

Chapter 6 deals with equivariance and gives the general theory of constructing MRE decision rules including a concise exposition of the relevant theory of Haar measure and application to MRE estimators for location and scale problems and UMPI test of linear hypothesis in the normal case besides equivalence of P -values, conditional confidence coefficient and posterior probability of hypothesis in an invariant setup.

Chapter 7 deals with large sample theory and gives the basics of stochastic small and large orders, delta method, asymptotic distribution of quantiles, consistency and asymptotic normality of MLE, M -estimators and posterior distribution, asymptotic agreement of predictive distribution with different priors and asymptotic distribution of LR test and chi-squared test of goodness of fit. This chapter also includes the theory of Laplace approximation to the posterior distribution.

Chapter 8 deals with hierarchical models and illustrates normal linear, Bernoulli, Poisson and mixture models with application to data allowing outliers, Bayesian robustness with respect to changes in the prior distribution of parameters as well as occurrences of unexpected observations. The presentation of the whole topic is essentially Bayesian but includes the use of empirical Bayes approach in detail in the case of normal linear models. Classical approach is mentioned but not pursued. This chapter also includes the theory of successive substitution sampling for approximating the posterior distribution.

Chapter 9 deals with sequential analysis and gives the general theory of SPRT including Bayes character and minimum expected sample size property, sequential set estimation including Stein's two stage rule and discussions on the relevance of stopping rules.

The book is self contained with background materials presented in appendices and includes a large variety of problems listed sectionwise at the end of each chapter including appendices A and B to enhance the understanding of the material dealt in the respective sections. Appendix A on measure and integration and Appendix B on probability are complete with proofs and provide excellent short courses on these topics. Appendix C gives a list of results on real, complex and functional analysis and Appendix D gives a list of standard and derived distributions. There are a few typographical errors but these can be easily corrected by readers.

Special features of the book are application of classical and Bayesian approaches side by side under a topic and critical appraisals of various results with examples and counter-examples. Other significant features are conciseness and originality of presentation besides being mathematically rigorous. In spite of the natural overlap with other standard textbooks, this book will be useful to all teachers, researchers and graduate students of theoretical statistics.