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The Glejser Test and the Median Regression

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Abstract

The Glejser test is affected by a non-vanishing estimation effect in the presence of skewness. We show that such effect occurs with contaminated errors as well, and that skewness correction is inappropriate when there is contamination. We consider the use of residuals estimated with conditional median regression (least absolute deviations or LAD) and discuss why LAD is effective against both skewness and contamination. The effectiveness of LAD is confirmed by simulations. With contaminated errors, both standard and skewness corrected Glejser tests perform poorly when based on least squares residuals. However, they perform very well when implemented using LAD residuals. The latter turns out to be a good alternative to bootstrap methods, which is generally used to solve the discrepancy between asymptotic and finite sample behaviour of a test.

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1 Introduction

The Glejser test is a well-known test for heteroscedasticity (Glejser, 1969), which is based on weak assumptions and is very easy to implement. It checks for the presence of a systematic pattern in the variances of the errors by estimating an auxiliary regression, where the absolute value of the residuals of the main equation is the dependent variable. Godfrey (1996) shows that this test is affected by an "estimation effect", namely it over-rejects the null hypothesis of homoscedasticity in the presence of skewed error distributions, and this effect does not vanish asymptotically.[1]

In addition to the above estimation effect in the Glejser test due to skewness, this paper shows the following.

[1] Im (2000), and independently Machado and Santos Silva (2000), propose a skewness corrected Glejser test, and we will discuss them in the following section.

- Godfrey's asymptotically non-vanishing estimation effect exists not only with skewed error distributions, but also with contaminated errors, which are symmetric and have tails heavier than normal. They are characterized by a greater probability of generating outliers, which affect the estimated coefficients and the residuals of the main equation, and thus the behaviour of the Glejser test.

- The use of robust residuals in the test function allows controlling contamination. Pagan and Hall (1983) stress the relevance of robust residuals for testing purposes. In particular, they suggest to estimate the main equation using OLS and to consider, for testing purposes, a robust estimator in the auxiliary regression. Instead, we propose and justify a different strategy, which involves robust estimation of the main equation and use of the robust residuals to implement the diagnostic test. Once the residuals are robustly computed, the need to estimate the auxiliary regression robustly is marginal.

- Among all the robust estimators, the least absolute deviation (LAD) is the most suitable one to estimate the residuals needed to implement the Glejser test. We show that the normal equation defining the LAD estimator is part of the Taylor expansion approximating the Glejser test function. When we use a different robust estimator, this result no longer holds, and the degree of skewness affects the behaviour of the test. When we use ordinary least squares (OLS) or any other non-robust estimator, both of skewness and contamination affect the behaviour of the test.

- The Glejser test computed with LAD residuals is asymptotically distributed as a χ^2.

The Monte Carlo experiments yield results in line with most of Godfrey's findings. There is over-rejection of the null hypothesis in the Glejser test with skewed error distributions; and under-rejection of the same under normality when the skewness-corrected Glejser test is used.

We include as a benchmark the Koenker test for heteroscedasticity, which is widely used and easy to implement. It requires the estimation of an auxiliary regression, where the dependent variable is the squared residual of the main equation, rather than the absolute value as in Glejser test.

In our simulations, the Koenker test turns out to have good size in the presence of skewness and contamination, when there is only one explanatory variable. However, the OLS based Koenker test is undersized in the case of normality[2] and over-rejects the true null also in the case of skewed and contaminated errors, when there are many explanatory variables. In addition, our results show the following.

- The Glejser test over-rejects the true null also in case of error contamination.
- The Glejser test is well behaved with asymmetric and contaminated errors if we use LAD instead of OLS residuals in the auxiliary regression. When the test is computed using LAD residuals, any skewness correction, as defined in Im (2000) or Machado and Santos Silva (2000), is redundant.
- The skewness corrected Glejser test, as the Koenker test, works properly with skewness and contamination when there is only one explanatory variable. However, with three or more explanatory variables both tests over-reject the true null in many experiments with skewed errors and in all the experiments with contaminated errors.
- The skewness corrected test and the Koenker test, just as the simple Glejser test, work fine with contamination, when there are many explanatory variables, if they are computed using LAD instead of OLS residuals.
- The presence of contamination in the explanatory variables causes some over-rejection in the LAD based tests, and this occurs because LAD is not designed to deal with outliers in the x's.

Finally Godfrey and Orme (1999), analysing the most popular tests of heteroscedasticity, find that nominal and finite sample critical values are substantially different. By using bootstrap methods they get good results, although there is some over-rejection in the Glejser test with asymmetric error distributions. Horowitz and Savin (2000) also consider bootstrap to obtain a good estimator of the critical value. Our paper shows that good results can be obtained using LAD residuals in the test functions. The LAD based tests have empirical sizes close to, and in some cases better than, the bootstrap estimates. With respect to the bootstrap, LAD is easier and faster to estimate since it is computationally less intensive.[3]

[2]This could be linked to the results by Tse (2002). The author shows that in case of test functions based on estimated regressors, OLS leads to the under rejection of the null.

[3]Indeed, quantile regression estimators are included in standard statistical packages and do not require specific programming nor a large number of replicates.

Section 2 defines the model and reviews the different versions of the Glejser test. Section 3 discusses Taylor expansion and its connection with the estimation effect. Section 4 summarizes the test functions and presents the asymptotic distribution of the Glejser test based on LAD residuals. Section 5 outlines the Monte Carlo experiments, and Sections 6 and 7 discuss the results in terms of the size and the power. The last section draws the conclusions.

2 Review of the Literature

Consider the linear regression model $y_t = x_t\beta + u_t$ where x_t is a $(1, k)$ vector containing the t-observation for all the k explanatory variables, β is a $(k, 1)$ vector of unknown parameters. Here, the u_t's, under the null, are i.i.d. errors with common distribution F, density f, zero median and constant variance σ^2. The vector b is a consistent estimator of β, $\hat{u}_t = y_t - x_t b_{OLS}$ is the OLS residual at time t, while $\hat{e}_t = y_t - x_t b_{LAD}$ is the LAD residual at time t. Tests for heteroscedasticity based on artificial regressions define an auxiliary equation $g(\hat{u}_t) = \alpha_0 + z_t\alpha$, where α and z_t are q-dimensional vectors. The Glejser test (1969) considers the case of $q = 1, g(\hat{u}_t) = |\hat{u}_t|, z_t = x_{ij}^r, r = \pm 0.5, \pm 1$. The hypotheses being tested are:

$$\begin{aligned} H_0 :& \quad E(u_t^2) = \sigma^2 = \alpha_0, \\ H_1 :& \quad E(u_t^2) = \sigma_t^2 = \alpha_0 + z_t\alpha. \end{aligned}$$

We assume the following regularity conditions for the explanatory variables x_t and z_t:

$$\begin{aligned} a1) &\quad n^{-1}\sum_t x_t' x_t \to \Omega_{xx}, \text{ positive definite;} \\ a2) &\quad n^{-1}\sum_t (z_t - \bar{z})'(z_t - \bar{z}) \to \Omega_{zz}, \text{ positive definite;} \\ a3) &\quad n^{-1}\sum_t (z_t - \bar{z})' x_t \to \Omega_{zx}. \end{aligned}$$

In the Glejser test, under the null hypothesis of homoscedasticity, the term nR^2 from the artificial regression, estimated with OLS, is assumed to be asymptotically distributed as a $\chi^2_{(q)}$, where n is the sample size and R^2 the coefficient of determination of the auxiliary regression. A large value of nR^2

leads to the rejection of the null.[4] The test is a check for orthogonality between the OLS residuals and z_t, i.e., on the significance of $T(b_{OLS}) = n^{-1/2}\sum_t (z_t - \bar{z})'|\hat{u}_i|$. Godfrey (1996) considers Taylor expansion of $T(b_{OLS})$,

$$T(b_{OLS}) = T(\beta_{OLS}) + n^{-1/2}\frac{\partial T(\beta_{OLS})}{\partial \beta_{OLS}} + n^{1/2}(b_{OLS} - \beta_{OLS}) + o_p(1),$$

to evaluate the robustness of the test. But, since "differentiability conditions are violated and standard Taylor series approach is not available", instead of using Taylor expansion, Godfrey approximates the test function $T(b_{OLS})$ with the expression:

$$T(b_{OLS}) = T(\beta_{OLS}) + (1 - 2p^*)\Omega_{zx} n^{1/2}(b_{OLS} - \beta_{OLS}) + o_p(1), \qquad (1)$$

where p^* is the probability of non-negative errors, and Ω_{zx} is the limit of the sample covariance matrix $n^{-1}\sum_t (z_t - \bar{z})' x_t$. The second term of equation (1) can be ignored if $p^* = 0.5$ (e.g., when the error distribution is symmetric), even if $\Omega_{zx} \neq 0$ which is often the case. This leads Godfrey to conclude the inadequacy of the Glejser test in the presence of skewness due to estimation effects.

Im (2000) and Machado and Santos Silva (2000) independently present modified Glejser tests to correct the skewness of the error distribution. Im (2000) approximates the Glejser test by

$$\begin{aligned}
T(b_{OLS}) - T(\beta_{OLS}) &= n^{-1/2}\sum_t (z_t - \bar{z})'(|\hat{u}_t| - |u_t|) \\
&= n^{-1/2}\left[\sum_{t\in n_1}(z_t - \bar{z})'(\hat{u}_t - u_t)\right. \\
&\qquad \left. - \sum_{t\in n_2}(z_t - \bar{z})'(\hat{u}_t - u_t)\right] + o_p(1) \\
&= -mn^{-1}\sum_t (z_t - \bar{z})' x_t n^{1/2}(b_{OLS} - \beta_{OLS}) + o_p(1), \qquad (2)
\end{aligned}$$

where the skewness coefficient $m = 2Pr(u_t \geq 0) - 1$ is estimated by $\hat{m} = \frac{n_1 - n_2}{n}$, the difference between the number of positive residuals n_1 and of negative residuals n_2 over the sample size n. Im (2000) proposes to implement

[4] When deciding between the null and the alternative hypothesis, we are not interested in estimating σ^2 per se. The error variance is a nuisance parameter, and we need to check if this parameter is constant over the sample since, under the unaccounted alternative of heteroscedasticity, any estimator of the vector of regression coefficient β is inefficient.

the artificial regression

$$|\hat{u}_t| - \hat{m}\hat{u}_t - \underline{\hat{\mu}} = \alpha_0 + z_t\alpha, \tag{3}$$

where $\underline{\hat{\mu}}$ is an estimate of $\underline{\mu} = E|u_t|$. Under the null, nR^2 from the artificial regression in (3) is asymptotically distributed as a $\chi^2_{(q)}$, and the result is not affected by the degree of skewness of the error distribution.

Machado and Santos Silva (2000) suggest to replace the dependent variable of the artificial regression$|\hat{u}_t|$ with $\hat{u}_t[I(\hat{u}_t \geq 0) - \eta]$, where $I()$ is the indicator function of the event in parenthesis, and η, defined as $\eta = Pr(u_t \geq 0)$, is estimated by $\hat{\eta} = 1/n \sum_t I(\hat{u}_t \geq 0)$.[5] The artificial regression becomes

$$\hat{u}_t[I(\hat{u}_t \geq 0) - \hat{\eta}] = \alpha_0 + z_t\alpha. \tag{4}$$

Finally, we consider another frequently implemented test, presented by Koenker (1981), which selects $g(\hat{u}_t) = \hat{u}_t^2$ as the dependent variable of the artificial regression:

$$\hat{u}_t^2 = \alpha_0 + z_t\alpha. \tag{5}$$

With i.i.d. errors having zero mean, finite variance, and finite fourth moment, under the null, nR^2 of equation (5) is assumed to be asymptotically distributed as a $\chi^2_{(q)}$. The test function $T(b_{OLS}) = n^{-1/2}\sum_t (z_t - \bar{z})'\hat{u}_t^2$ checks the orthogonality between z_t and the squared residuals, and it does not violate the differentiability conditions, since

$$\text{plim } n^{-1/2}\frac{\partial T(\beta_{OLS})}{\partial \beta_{OLS}} = -2\text{plim } n^{-1}\sum_t (z_t - \bar{z})' x_t u_t = 0.$$

This allows Godfrey (1996) to asses the robustness of the Koenker test with respect to estimation effects. However, in their simulations, Godfrey and Orme (1999) find that the robustness of this test decreases when the number of test variables increases, and we will relate this result to the lack of robustness of the residuals of the main equation.

3 Estimation Effect in Taylor Expansions and Asymptotic Approximations

We propose to introduce the following approximations in the Taylor expansion.

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[5]Since the Im test and the Machado and Santos Silva test are numerically identical, in the following sections we focus the discussion on the Im test.

(i) In view of the non-differentiability of the absolute value function of the Glejser test, we choose to use the sign( ) function in place of the derivative. In other words, we define $\frac{\partial T(\beta)}{\partial \beta} = \psi(u_t)$, where $\psi(\ ) =$ sign( ), which is the directional derivative of the absolute value function.[6] Our sign function $\psi(\ )$ replaces

- the term $(1 - 2p^*)$ of equation (1) (Godfrey, 1996),
- the sample splitting $\hat{m} = \frac{n_1 - n_2}{n}$ of equation (3) (Im, 2000), or
- the indicator function $I(\hat{u}_t \geq 0 - \hat{\eta})$ in the artificial regression (4) (Machado and Santos Silva, 2000).

Since $\psi(\ )$ enters the normal equations defining the LAD estimator, this explains why LAD is the "natural choice of conditional location for the equivalence of $T(b)$ and $T(\beta)$ to hold under general conditions" (Machado and Santos Silva, 2000). In LAD, the number of positive residuals balances the number of negative residuals by construction, since the LAD fitted plane computes the conditional median of the y's. In OLS, this is not necessarily the case, since the OLS fitted plane estimates the conditional mean of the y's. In the case of asymmetric distributions, the mean differs from the median leading to a number of positive residuals larger or smaller than the number of negative ones, according to the presence of positive or negative skewness.

(ii) Both in Taylor expansion and Godfrey asymptotic approximation in (1), by considering β as a functional, we replace $n^{1/2}(b - \beta)$ by its asymptotic representation, i.e., the influence function (IF), provided the required regularity conditions are fulfilled. The IF is the derivative of the selected estimator with respect to contamination

$$IF(b, F, y) = \left. \frac{\partial b((1-\lambda)F + \lambda\delta_y)}{\partial \lambda} \right|_{\lambda=0}$$

(Hampel et al., 1986). In this derivative, the empirical distribution is expressed as the linear combination of F, the assumed distribution, and δ_y, the probability measure assigning unit mass at point y to account for the presence of an outlying observation in y. The term λ defines the degree of contamination of the empirical distribution, i.e., the percentage of outliers in the data. Therefore $n^{1/2}(b - \beta) = n^{-1/2} \sum_t IF(b, F, x_t, y_t) + o_p(1)$ defines the impact of the degree of

[6]Technically, at $u_t = 0$ the $|u_t|$ function has no derivative. However, the directional derivative is a generally accepted approximation.

contamination λ on the regression coefficients and, through them, on the test function $T(\beta)$.

Considering (i), (ii), Taylor expansion of the Glejser test $T(b_{OLS}) = n^{-1/2}\sum_t (z_t - \bar{z})'|\hat{u}_t|$ becomes:

$$\begin{aligned}
&T(b_{OLS}) - T(\beta_{OLS}) \\
&= n^{-1/2}\frac{\partial T(b_{OLS})}{\partial \beta_{OLS}} n^{-1/2}(T(b_{OLS} - \beta_{OLS}) + o_p(1) \\
&= n^{-1}\sum_t \psi(u_t(\beta_{OLS}))(z_t - \bar{z})' x_t n^{-1/2}\sum_t IF(b_{OLS}, F, x_t, y_t) + o_p(1) \\
&= n^{-1}\sum_t \psi(u_t(\beta_{OLS}))(z_t - \bar{z}) x_t n^{-1/2}\sum_t (Ex_t' x_t)^{-1} u_t(\beta_{OLS}) x_t' + o_p(1). \quad (6)
\end{aligned}$$

Equation (6) shows that the estimation effect depends upon the degree of skewness (which is only one of possible imbalances) measured by the sign function and is affected by the lack of boundedness in the influence function of b_{OLS}. Outliers can cause inconsistency since $IF(b_{OLS}, F, x_t, y_t)$ is not bounded in the errors: $n^{-1/2}(b_{OLS} - \beta_{OLS}) \neq O_p(1)$ and OLS is not the appropriate estimator in the presence of outliers. Analogously, the impact of outliers can be seen in Godfrey's asymptotic approximation in (1):

$$\begin{aligned}
T(b_{OLS}) &= T(\beta_{OLS}) + (1 - 2p^*)\Omega_{zx} N^{1/2}(b_{OLS} - \beta_{OLS}) + o_p(1) \\
&= T(\beta_{OLS}) + (1 - 2p^*)\Omega_{zx} n^{-1/2}\sum_t IF(b_{OLS}, F, x_t, y_t) + o_p(1) \\
&= T(\beta_{OLS}) + (1 - 2p^*)\Omega_{zx} n^{-1/2}\sum_t (Ex_t' x_t)^{-1} u_t(\beta_{OLS}) x_t' + o_p(1). \quad (7)
\end{aligned}$$

Once again, by replacing the term $n^{1/2}(b_{OLS} - \beta_{OLS})$ with the influence function of b_{OLS}, the impact of contamination on the Glejser test is self-evident. Godfrey shows the existence of an estimation effect when working on the residuals, while this effect is irrelevant when the true errors are in use. We relate this result to the lack of robustness of the OLS residuals.

The impact of error and/or data contamination does not vanish asymptotically, and can be controlled only by implementing a robust estimator, which is characterized by a bounded IF. When we select a robust estimator for β, $n^{1/2}(b_{robust} - \beta) = n^{-1/2}\sum_t IF(b_{robust}, F, x_t, y_t) + o_p(1)$ is bounded, asymptotically normal with zero mean and variance $E(IF)^2$, and the effect of contamination is under control. In particular, when we consider the LAD estimator, $n^{1/2}(b_{LAD} - \beta_{LAD}) = n^{-1/2}\sum_t IF(b_{LAD}, F, x_t, y_t) + O_p(n^{-1/4})) =$

$n^{-1/2} \sum_t \frac{(Ex_t' x_t)^{-1} \psi(u_t(\beta_{LAD}))x_t'}{2f(0)} + O_p(n^{-1/4})$, where $f(0)$ is the height of the error density at the median. The LAD estimator is asymptotically normal, i.e., $n^{1/2}(b_{LAD} - \beta_{LAD}) \sim N(0, (2f(0))^{-2}\Omega_{xx}^{-1})$ (Koenker and Bassett, 1982). Its influence function $IF(b_{LAD}, F, x_t, y_t)$ is bounded with respect to outlying errors,[7] and by construction, the positive and negative residual are balanced. This makes any adjustment in the standard Glejser test unnecessary. Indeed, the Taylor expansion of the Glejser test based on LAD residuals $T(b_{LAD}) = n^{-1/2} \sum_t (z_t - \bar{z})' |\hat{e}_t|$ is:

$$\begin{aligned} T(b_{LAD}) &= T(\beta_{LAD}) \\ &\quad + n^{-1} \sum_t \psi(u_t(\beta_{LAD}))(z_t - \bar{z})' x_t n^{-1/2} \sum_t \frac{(Ex_t' x_t)^{-1} \psi(u_t(\beta_{LAD}))x_t'}{2f(0)} \\ &\quad + \text{remainder}, \end{aligned} \tag{8}$$

which shows the robustness of the LAD based Glejser test with respect to both skewness and contamination, without the introduction of any correcting factor.

Summarizing, equations (6) and (7) show that both asymmetry and contamination can affect the behaviour of the Glejser test.

- With OLS, the skewness coefficient p^* in equation (7) is free to take any value. In terms of Taylor expansion (6), the sign function $\psi(u_t(\beta_{OLS}))$ is free to take any number of positive (or negative) values, since the OLS residuals are not necessarily balanced between positive and negative values. This is not the case in the LAD regression, where the number of positive residuals offsets the number of negative ones by construction.

- With OLS, the influence function $IF(\beta_{OLS}, F, x_t, y_t)$ is not bounded so that large residuals and/or outliers in the x's can cause a drastic change in the vector of the estimated coefficients b_{OLS}. This is not the case for $IF(b_{LAD}, F, x_t, y_t)$, which is bounded with respect to large residuals (but not with respect to outliers in the x's).

4 Asymptotic Distribution of the LAD Test Function

To compare the heteroscedasticity tests defined in Section 2, the selected linear regression model is estimated by OLS and LAD. We have defined

[7] However LAD is not bounded with respect to large x's, which are unconstrained within the influence function. When using the OLS estimator $\psi(u_t(\beta_{OLS})) = u_t(\beta_{OLS})$, the sign function in the IF is not there, and both the errors and the x's are unbounded.

$\hat{u}_t = y_t - x_t\ b_{OLS}$ and $\hat{e}_t = y_t - x_t\ b_{LAD}$, which allow to implement the following six artificial regressions,[8] corresponding to six different tests of heteroscedasticity:

$$\begin{array}{llll} a) & |\hat{u}_t| & = a_0 + a z_t, & (G_{OLS}) \\ b) & |\hat{u}_t| - \hat{m}\hat{u}_t - \underline{\hat{\mu}} & = a_0 + a z_t & (Im_{OLS}) \\ c) & \hat{u}_t^2 & = a_0 + a z_t, & (K_{OLS}) \\ d) & |\hat{e}_t| & = a_0 + a z_t, & (G_{LAD}) \\ e) & |\hat{e}_t| - \hat{m}\hat{e}_t - \underline{\hat{\mu}} & = a_0 + a z_t & (Im_{LAD}) \\ f) & \hat{e}_t^2 & = a_0 + a z_t, & (K_{LAD}) \end{array}$$

Equation (a) is the standard Glejser test (Glejser, 1969) implemented with OLS residuals (G_{OLS}), while equation (d) is a Glejser test based on LAD residuals (G_{LAD}). In (b), we consider the skewness adjusted Glejser test (Im, 2000), computed using OLS residuals (Im_{OLS}). In equation (e), we define the same test using LAD residuals (Im_{LAD}). Equation (c) is the Koenker test (Koenker, 1981) based on the squared OLS residuals (K_{OLS}). In (f), the same test is computed using squared LAD residuals (K_{LAD}).

In the six auxiliary equations (a)–(f), the term nR^2 is asymptotically distributed as a $\chi^2_{(q)}$, under the null hypothesis of i.i.d. errors. Indeed, the distribution of the LAD based tests (d), (e) and (f) can be obtained as:

$$\begin{aligned} T(b_{LAD}) &= n^{-1/2}\sum_t (z_t - \bar{z})'|y_t - x_t b_{LAD}| \\ &= n^{-1/2}\sum_t (z_t - \bar{z})'|y_t - x_t\beta_{LAD} + x_t(\beta_{LAD} - b_{LAD})| \\ &= n^{-1/2}\sum_t (z_t - \bar{z})'|y_t - x_t\beta_{LAD}| \\ &\quad -n^{-1/2}\sum_t (z_t - \bar{z})' x_t \psi(u_t(\beta_{LAD}))(b_{LAD} - \beta_{LAD}) + o_p(1) \\ &= T(\beta_{LAD}) - n^{-1}\sum_t (z_t - \bar{z})' x_t \psi(u_t(\beta_{LAD})) O_p(1) + o_p(1) \\ &= T(\beta_{LAD}) + o_p(1) \qquad (9) \end{aligned}$$

where the last equality follows from the fact that the term $n^{-1}\sum_t (z_t - \bar{z})' x_t \psi(u_t(\beta_{LAD})) O_p(1)$, can be merged with the $o_p(1)$ remainder, which is

[8] All the artificial regressions are estimated by OLS. The use of LAD in the auxiliary equations turns out to be of little help in our simulations, once the residuals from the main equation are robustly computed (these results are not reported).

possible for median regression.[9] The expansion in (9) is valid for any consistent estimator of β, but with skewed and contaminated errors, the last equality holds only for residuals computed from the median regression.[10] By the Central Limit Theorem, under the null hypothesis of homoscedasticity, $T(b_{LAD})$ is asymptotically normal, and nR^2 from the auxiliary regressions (d), (e) and (f) are asymptotically distributed as $\chi^2_{(q)}$.

5 Plan of the Experiments

The Monte Carlo study here implemented, far from being exhaustive, replicates some of the experiments presented in other simulation studies on the Glejser test.[11] The data generating process is $y_t = constant + \beta x_t + u_t$, where $constant = 0.3$ and $\beta = 0.6$. The x's are generated from a uniform distribution in the range (0,1), x_{1t}; from a log-normal, Λ(3,1), where $\log(x_{2t})$ follows a $N(3,1)$; from a chi-square with one degree of freedom, x_{3t}; from a contaminated normal, designed to have 90% of the observations generated by a standard normal and the remaining 10% generated by a zero mean normal with variance equal to 100, x_{4t}; from a student-t with two degrees of freedom, x_{5t}. We begin with a simple model with only one explanatory variable and then we increase the number of explanatory variables in the regression. Godfrey and Orme (1999) stress the relevance of checking the behaviour of a test by employing multiple regression models, since "results obtained from simple experimental designs may be an unreliable guide to finite sample performance". Thus we gradually increase the number of explanatory variables both in the main regression and in the equation defining heteroscedasticity. This allows checking the stability of the results with respect to the increasing complexity of a model. The errors are serially independent and independent of the x_t's. We consider the following error distributions: standard normal; chi-square with two degrees of freedom; log-normal, Λ(0,1); student-t with five degrees of freedom; contaminated normal, where 90% of the observations are generated by a standard normal and the remaining 10% are generated by a zero mean normal with variance equal to 100, CN(10%,100).

[9] Equation (9) is slightly different from the asymptotic distribution of the Machado and Santos Silva (2000) test function. They define a skewness correcting factor which we show to be unnecessary when implementing LAD.

[10] Equation (9), by replacing the absolute value with the squared residuals, holds for the test (f) as well, due to the use of LAD residuals, $u_t(\beta_{LAD})$, in the auxiliary equation.

[11] The plan of the experiments takes into account some of the distributions and the models analysed in the simulations implemented by Godfrey (1996), Godfrey and Orme (1999), Im (2000), and Machado and Santos Silva (2000).

The chi-square and the log-normal distributions are used analyse the behaviour of the tests in the presence of skewness, while the student-t and the contaminated distributions are characterized by thick tails, and induce symmetrically distributed outliers. All distributions, with the sole exception of the contaminated normal, are taken from other Monte Carlo studies. Our contribution is to introduce simulation experiments with contaminated distribution both in the errors and in the explanatory variables in order to verify the impact of outliers on the selected tests.

We consider two sample sizes, n=34 and n=80. The number of replicates for each experiment is 10 000. We select as test variable $z_t = x_t$.

Under the alternative, the errors are defined as $\sigma_t u_t$, with $\sigma_t = [0.2\{1 + az_t^2/var(z_t)\}]^{1/2}$, where z_t^2 is defined as a single variable in some models and as a vector of variables in others. Accordingly, a is a single coefficient, and is set equal to 8, or is a vector of coefficients, each one equal to 8.[12]

Unlike previous simulation studies, in our study, we introduce contamination in the errors and in one of the explanatory variables; we extend the LAD approach to the modified Glejser and to the Koenker test; we consider both simple and multivariate regression models.

6 Results: Size

Table 1 reports the rejection rates of the tests under the null hypothesis of homoscedasticity, at the 5% nominal level. In the table, the first column gives the list of the tests analysed here, where the first three tests are the OLS based ones, and the remaining three are the LAD based tests. All the other columns report the rejection rates of these tests under different error distributions. The table is further divided into eleven parts, I to XI, each defining the different distributions generating the test variables x_t: uniform in the first part, x_{1t}, log-normal in the second, x_{2t}, chi-square in the third part, x_{3t}, contaminated normal in the fourth, x_{4t}, and student-t in the fifth, x_{5t}. In these five parts, the term nR^2 of the auxiliary regressions (a) through (f), under the null is assumed to be distributed as a $\chi^2_{(1)}$ with critical value of 3.84 at the 5% nominal level. The remaining parts of the table present models having more than one explanatory variable. In parts VI to VIII we analyse a model with three explanatory variables, $y_t = 0.3+0.6x_{1t}+0.6x_{2t}+0.6x_{jt}+u_t$ with j=3,4,5, and the test statistics are assumed to be distributed as $\chi^2_{(3)}$ with critical value of 7.81 at the 5% level. Then we consider models with four explanatory variables, $y_t = 0.3+0.6x_{1t}+0.6x_{2t}+0.6x_{3t}+0.6x_{jt}+u_t$

[12] All the simulations are implemented using Stata, version 6.

with j=4,5, and the term nR^2 is assumed to be distributed as a $\chi^2_{(4)}$ with a critical value of 9.49. Finally, in the last part, we take into account a model comprising all five explanatory variables, $y_t = 0.3 + 0.6x_{1t} + 0.6x_{2t} + 0.6x_{3t} + 0.6x_{4t} + 0.6x_{5t} + u_t$. The test statistics are assumed to follow $\chi^2_{(5)}$ with critical value of 11.1 at the 5% level.

To judge about the size of the tests we take into account the criteria in Godfrey (1996): rejection rates in the range of 3.5%-6.6% can be considered "good", while those falling in the range 2.6%-7.6% are "adequate". In Table 1, we report in bold the over-rejections. The tests in this table are computed using estimated residuals.

Table 1 shows the following when using OLS residuals in the test function.

(i) The Glejser test over-rejects the null not only with skewness but also with contamination. This occurs in all the parts of the table, and the first line of each part is in bold everywhere but with $N(0,1)$ errors and with contaminated x's. Indeed, in part IV the problem shared by all the OLS based tests seems to be under-rejection.[13]

(ii) In the model with one explanatory variable, the OLS based Im test improves upon the OLS based Glejser test yielding generally good rejection rates. However, it tends to over-reject the null with error contamination, when the explanatory variable follows a chi-square distribution (part III). With skewness, Λ(0,1), when the x's are drawn from a uniform and the sample is small (part I). In addition, the Im and the Koenker tests strongly under-reject the true null when x_t is contaminated, with all the error distributions (part IV) and in both sample sizes, showing a tendency to under-reject even when the x's come from a student-t distribution (part V).[14]

[13] In comparing our results with the ones in Godfrey (G(1.0) test, p. 287, 1996), we get very similar empirical sizes, coinciding up to the second digit, for the Glejser test in case of log-normal and chi-squared errors. The main difference is in the experiments with student-t errors, where we find over-rejection in the Glejser test, and this is the case in the contaminated errors as well. The latter experiment is not analysed in Godfrey nor in other simulation studies on the Glejser test.

[14] The discrepancies between nominal and empirical sizes for the Im test reported in our simulations occur in experiments not implemented in Im (2000) or in other Monte Carlo studies.

TABLE 1. GLEJSER TESTS USING THE ESTIMATED RESIDUALS, $n = 34, 80$, OLS IN THE AUXILIARY REGRESSION, 10000 REPLICATES, $H_0 : \sigma_t = \sigma$

I: x_{1t} DRAWN FROM A UNIFORM $(0, 1)$

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.055	.051	**.137**	**.137**	**.117**	**.121**	**.184**	**.182**	**.116**	**.084**
Im_{OLS}	.048	.048	.066	.058	.064	.056	**.070**	.054	.056	.049
K_{OLS}	.049	.050	.066	.059	.063	.051	.064	.045	.044	.041
G_{LAD}	.040	.045	.047	.050	.047	.048	.049	.045	.039	.045
Im_{LAD}	.040	.044	.052	.053	.054	.051	.054	.047	.039	.045
K_{LAD}	.041	.048	.031	.033	.033	.032	.030	.028	.029	.036

II: x_{2t} DRAWN FROM A LOG-NORMAL (3,1)

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.039	.039	**.080**	**.083**	**.070**	**.072**	**.094**	**.092**	**.079**	**.074**
Im_{OLS}	.033	.037	.039	.035	.038	.034	.044	.038	.060	.059
K_{OLS}	.024	.028	.041	.035	.042	.035	.049	.041	.049	.040
G_{LAD}	.045	.048	.034	. 035	.040	.037	.040	.035	.050	.044
Im_{LAD}	.044	.048	.038	.037	.043	.039	.043	.036	.049	.044
K_{LAD}	.038	.040	.036	.033	.039	.034	.043	.038	.050	.045

III: x_{3t} DRAWN FROM A $\chi^2_{(1)}$

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.043	.046	**.092**	**.103**	**.079**	**.091**	**.102**	**.116**	**.089**	**.086**
Im_{OLS}	.037	.043	.046	.042	.046	.043	.047	.046	**.068**	**.070**
K_{OLS}	.028	.034	.047	.042	.045	.045	.050	.049	.056	.048
G_{LAD}	.044	.052	.039	.041	.040	.043	.041	.044	.050	.047
Im_{LAD}	.044	.051	.045	.043	.045	.046	.044	.046	.050	.047
K_{LAD}	.043	.044	.039	.037	.041	.041	.044	.045	.056	.053

IV: x_{4t} DRAWN FROM A CONTAMINATED NORMAL (10%,100)

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.023	.028	.042	.052	.034	.051	.048	.060	.041	.040
Im_{OLS}	.019	.026	.021	.022	.019	.026	.023	.023	.029	.032
K_{OLS}	.017	.024	.027	.030	.024	.037	.030	.042	.032	.034
G_{LAD}	**.069**	**.079**	.050	.053	.051	.057	.045	.054	.057	.059
Im_{LAD}	**.068**	**.079**	.054	.054	.056	.059	.047	.055	.056	.059
K_{LAD}	.054	**.070**	.044	.047	.042	.052	.044	.056	.052	.058

TABLE 1 (CONTINUED)

V: x_{5t} DRAWN FROM A $t_{(2)}$

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.039	.037	**.073**	**.072**	**.068**	.065	**.088**	**.081**	**.073**	**.067**
Im_{OLS}	.034	.033	.036	.031	.038	.031	.041	.034	.055	.053
K_{OLS}	.025	.023	.037	.030	.035	.030	.042	.036	.043	.034
G_{LAD}	.043	.049	.034	.033	.037	.033	.034	.031	.044	.040
Im_{LAD}	.043	.048	.038	.035	.041	.034	.037	.032	.044	.040
K_{LAD}	.034	.038	.030	.028	.034	.031	.038	.033	.047	.041

VI: x_{1t}, x_{2t} AND x_{3t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.039	.043	**.129**	**.152**	**.112**	**.121**	**.181**	**.197**	**.132**	**.119**
Im_{OLS}	.031	.038	.058	.050	.061	.049	**.077**	.057	**.086**	**.082**
K_{OLS}	.029	.035	.061	.053	.063	.052	**.078**	.061	**.069**	.055
G_{LAD}	.034	.046	.039	.046	.044	.045	.050	.051	.057	.057
Im_{LAD}	.034	.046	.048	.050	.051	.050	.057	.053	.056	.057
K_{LAD}	.044	.052	.043	.043	.047	.039	.052	.052	.063	.061

VII: x_{1t}, x_{2t} AND x_{4t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.030	.038	**.089**	**.127**	**.075**	**.104**	**.119**	**.161**	**.110**	**.088**
Im_{OLS}	.025	.035	.040	.040	.042	.040	.052	.044	**.081**	.058
K_{OLS}	.022	.031	.050	.047	.050	.045	.065	.052	**.072**	.047
G_{LAD}	.048	.062	.045	.053	.047	.055	.053	.055	**.073**	.064
Im_{LAD}	.047	.062	.052	.058	.054	.059	.056	.057	**.076**	.064
K_{LAD}	.053	.066	.053	.049	.054	.051	.061	.056	**.084**	.066

VIII: x_{1t}, x_{2t} AND x_{5t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.042	.038	**.121**	**.136**	**.103**	**.115**	**.166**	**.181**	**.118**	**.105**
Im_{OLS}	.032	.036	.056	.045	.052	.045	.067	.055	**.076**	**.072**
K_{OLS}	.029	.032	.056	.044	.055	.049	.065	.055	.062	.048
G_{LAD}	.036	.046	.042	.043	.040	.043	.043	.047	.052	.054
Im_{LAD}	.036	.046	.051	.048	.048	.046	.050	.049	.052	.055
K_{LAD}	.041	.051	.036	.036	.043	.042	.045	.047	.057	.056

TABLE 1 (CONTINUED)

IX: x_{1t}, x_{2t}, x_{3t} AND x_{4t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.033	.039	**.109**	**.142**	**.096**	**.116**	**.150**	**.186**	**.113**	**.105**
Im_{OLS}	.026	.035	.050	.046	.049	.047	.062	.050	**.073**	**.076**
K_{OLS}	.025	.031	.051	.052	.051	.053	.065	.064	.062	.058
G_{LAD}	.041	.057	.044	.055	.042	.052	.047	.056	.062	**.072**
Im_{LAD}	.040	.057	.057	.060	.051	.056	.057	.059	.064	**.072**
K_{LAD}	.051	.063	.044	.051	.042	.054	.053	.062	**.073**	**.079**

X: x_{1t}, x_{2t}, x_{3t} AND x_{5t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.040	.042	**.128**	**.147**	**.110**	**.125**	**.187**	**.205**	**.135**	**.123**
Im_{OLS}	.032	.037	.056	.044	.057	.053	**.077**	.061	**.090**	**.088**
K_{OLS}	.030	.032	.061	.051	.058	.053	**.079**	.067	**.076**	.056
G_{LAD}	.034	.046	.039	.042	.040	.046	.053	.052	.061	.062
Im_{LAD}	.033	.046	.048	.048	.047	.052	.061	.054	.060	.063
K_{LAD}	.043	.054	.040	.042	.044	.049	.054	.058	**.070**	**.070**

XI: $x_{1t}, x_{2t}, x_{3t}, x_{4t}$ AND x_{5t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$		$CN(10\%,100)$	
	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$	$n=34$	$n=80$
G_{OLS}	.029	.033	**.103**	**.137**	**.085**	**.119**	**.149**	**.193**	**.115**	**.116**
Im_{OLS}	.022	.030	.048	.039	.046	.046	.066	.054	**.083**	**.082**
K_{OLS}	.019	.027	.050	.048	.049	.053	**.070**	.066	**.073**	.061
G_{LAD}	.031	.056	.038	.049	.040	.050	.050	.057	**.069**	**.078**
Im_{LAD}	.029	.056	.048	.056	.050	.057	.058	.060	**.069**	**.077**
K_{LAD}	.044	**.070**	.044	.049	.048	.055	.058	**.069**	**.079**	**.087**

Note: Under H_0, the "good" rejection rates are in the 3.5%-6.6% range. The over-rejections are reported in bold.

(iii) When we consider the experiments with more than one explanatory variable, over-rejection occurs in the presence of contamination and of skewness for the OLS based Glejser test with both sample sizes and with all the combinations of the explanatory variables here considered. For the OLS based Im test, the over-rejection occurs in the presence of error contamination regardless of the sample size (parts VI to XI), while with log-normal errors and in small samples we find over-rejection in parts VI, VIII and X. The OLS based Koenker test over-rejects in small samples with error contamination, as can be seen in parts VI, VII, X and XI, and with log-normal errors in parts VI, X, XI. Indeed, both Im and Koenker have inadequate rates with log-normal errors in the small sample (parts VI, X), while Im presents inadequate rates with contaminated errors in parts VI, VII, X and XI.[15]

(iv) The Koenker test and, to a lower extent, the Im test under-reject the null in case of normality. In most of the experiments with normal errors the Koenker test has inadequate rates (parts II, IV, V, VII, IX, XI). The under-rejection tends to disappear when n increases in the Im but not in the Koenker test.[16]

(v) The OLS based Im test substantially improves upon the OLS based Glejser test, although it does not close the gap between actual and nominal size of the test when the errors are contaminated: in parts VI, VII, X, XI the Im test presents inadequate rates of rejections and in parts III, VIII and IX adequate, showing a tendency to over-reject with contaminated errors in seven out of eleven experiments.[17]

When using LAD residuals in the auxiliary regression the simulations show the following.

(ia) The Glejser LAD based test does not over-reject with skewness in all the experiments. With contamination, it shows a tendency to over-reject in part VII, IX, XI, all characterized by the presence of x_{4t}, that is of contamination in the explanatory variable. This is a consequence of the definition of LAD, which does not control contamination in the x's.[18]

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[15]Once again, the over-rejections of the Im test occur in experiments not implemented elsewhere. The result for the Koenker test agrees with the findings in Machado and Santos Silva (2000) and in Godfrey and Orme (1999). They point out the loss of robustness of the Koenker test when the number of explanatory variables increases, as can be seen in parts VI to XI of our Table 1.

[16]This result for the Koenker and Im tests agrees with Godfrey and Orme (1999) findings, and can be explained with the results in Tse (2002): in case of test functions based on estimated regressors, OLS leads to the under-rejection of the null.

[17]Error contamination has not been analysed in other Monte Carlo studies. The contaminated distribution considered in Godfrey and Orme (1999) is contaminated in the center of the distribution and provides a bimodal error distribution. It differs from the one here selected, which is contaminated in the tails and is currently used to model outliers.

[18]Machado and Santos Silva (2000) implement the LAD based Glejser test in their simulations, and find evidence of a good agreement between nominal and empirical size of this test, as in our case. Their second set of variables is similar to the one used in our simulations. However, they do not consider the use of LAD residuals in the Koenker test

(iia) With one explanatory variable, the Im and the Koenker LAD based tests work fine.

(iiia) With many explanatory variables, the Im and Koenker tests tend to over-reject the null only in those experiments including x_{4t}. However there are only two experiments with "inadequate" rates, for the Koenker test in parts VII and XI.

(iva) In the Koenker test, the under-rejection in the case of normality improves or disappears, with the sole exception of section IV, which once again considers the contaminated explanatory variable x_{4t}.

(va) In most of the experiments of part IV the LAD based tests improve upon the OLS based ones, although this estimator is not designed to deal with outliers in the x's.

Finally, in parts III, VI, VIII, X, and to a lesser extent in parts VII, IX, XI, the size distortion in the OLS-Glejser and the OLS-Im tests does not disappear when the sample increases, while it disappears (in parts III, VI, VII,VIII, X) or decreases (in parts IX and XI) when the LAD residuals are used to implement the same tests.

These results imply that the Glejser test in conjunction with LAD residuals does not need any correction for skewness and/or contamination. The use of LAD residuals generally improves upon the results obtained with OLS residuals. The correction introduced by Im (2000) and Machado and Santos Silva (2000) does not work when there is more than one explanatory variable and the test function is estimated using OLS residuals. This is surprising, since the correction is designed to solve the over-rejection problem of the Glejser test with skewed errors. When the explanatory variable is contaminated, all the OLS based tests are undersized regardless the error distribution. When there are as many as five explanatory variables, all the tests here analysed are undersized in the case of normality.

Godfrey and Orme (1999) in their paper propose the use of bootstrap to estimate the size of the Glejser type tests. The model we implement in parts VI, VII and VIII of table 1 are very similar to the model analysed in their article.[19] Therefore, in Table 2, we report their bootstrap results,

or in their skewness adjusted Glejser test. In addition, they do not analyse the impact of error contamination, which affects the behaviour of their test particularly in models with more than one explanatory variable.

[19]They take into account three explanatory variables that are realizations from a (1,31) uniform, a (3,1) log-normal, and AR(1) process with coefficient of 0.9. Their sample size is equal to 80.

and compare them with our results for the LAD based tests in the common experiments: standard normal, $\chi^2_{(2)}$, t_5 and log-normal errors. This table shows that the two sets of results are comparable, but bootstrap still causes over-rejections in the Glejser test with a $\chi^2_{(2)}$ distribution and with log-normal errors. However, the main advantage of using LAD over bootstrap is in terms of a reduced computational burden.

TABLE 2. EMPIRICAL SIZE COMPUTED USING BOOTSTRAP AND LAD, $n = 80$.

VI: x_{1t}, x_{2t} AND x_{3t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$	
	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap
Glejser	0.046	0.050	0.039	**0.067**	0.044	0.046	0050	**0.078**
Im	0.046	0.051	0.048	0.049	0.051	0.048	0.057	0.059
Koenker	0.052	0.052	0.041	0.052	0.047	0.050	0.052	0.057

VII: x_{1t}, x_{2t} AND x_{4t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$	
	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap
Glejser	0.048	0.050	0.045	**0.067**	0.047	0.046	0.053	**0.078**
Im	0.047	0.051	0.052	0.049	0.054	0.048	0.056	0.059
Koenker	0.053	0.052	0.053	0.052	0.054	0.050	0.061	0.057

VIII: x_{1t}, x_{2t} AND x_{5t} TOGETHER

	$N(0,1)$		$\chi^2_{(2)}$		t_5		$\Lambda(0,1)$	
	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap	LAD	bootstrap
Glejser	0.036	0.050	0.042	**0.067**	0.040	0.046	0.043	**0.078**
Im	0.036	0.051	0.051	0.049	0.048	0.048	0.050	0.059
Koenker	0.041	0.052	0.036	0.052	0.043	0.050	0.045	0.057

Note: The table compares our results in Table 1 (sections VI and VII) for the tests based on LAD residuals, with the results in Godfrey and Orme (1999) computed using bootstrap in the same experiments and the same sample size ($n = 80$). Under H_0, the "good" rejection rates are in the 3.5%-6.6% range. The "adequate" rejection rates are in the 2.6%-7.6%. All the over-rejections are reported in bold.

7 Results: Power

Table 3 shows the power of the Glejser tests in the small sample, $n = 34$. Once again, the table is divided into parts, I to X. The first five parts consider the model with only one explanatory variable, and the true alternative is $H_a : \sigma_t = [0.2\{1 + 8x^2_{jt}/var(x_{jt})\}]^{1/2}$ for j = 1,2,3,4,5 in turn. The remaining parts consider multivariate models. In parts VI and VII, the model has three explanatory variables and the alternative is $H_a : \sigma_t = [0.2\{1 + 8x^2_{1t}/var(x_{1t}) + 8x^2_{2t}/var(x_{2t}) + 8x^2_{jt}/var(x_{jt})\}]^{1/2}$ for j = 3,4. In parts VIII and IX, the model has four explanatory variables, and the true alternative is $H_a : \sigma_t = [0.2\{1+8x_{1t}^2/var(x_{1t})+8x_{2t}^2/var(x_{2t})+8x_{3t}^2/var(x_{3t})+8x_{jt}^2/var(x_{jt})\}]^{1/2}$ for j = 4,5. In the final part of the table, there are five explanatory variables in the main equation, and the true alternative is $H_a : \sigma_t = [0.2\{1+8x_{1t}^2/var(x_{1t})+8x_{2t}^2/var(x_{2t})+8x_{3t}^2/var(x_{3t})+8x_{4t}^2/var(x_{4t})+8x_{5t}^2/var(x_{5t})\}]^{1/2}$. The auxiliary regressions implemented are those in equations a) to f) and, according to the model analysed in each part, we estimate them by selecting the correct choice of explanatory variables.

In this table, the highest rates of the OLS based tests are reported in bold, while the highest rates of the LAD based tests are underlined.

TABLE 3. POWER OF THE GLEJSER TESTS,
$n = 34,\ H_a : \sigma_t = \{0.2[1 + 8x_t^2/var(x_t)]\}^{1/2}$

I: x_{1t} DRAWN FROM A UNIFORM (0,1)

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.926**	**0.809**	**0.823**	**0.648**	**0.360**
Im_{OLS}	0.913	0.643	0.680	0.417	0.257
K_{OLS}	0.835	0.494	0.532	0.283	0.150
G_{LAD}	0.916	0.659	0.715	0.435	0.273
Im_{LAD}	0.914	0.685	0.739	0.456	0.275
K_{LAD}	0.812	0.349	0.403	0.179	0.122

II: x_{2t} DRAWN FROM A LOG-NORMAL (3,1)

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.829**	**0.726**	**0.742**	**0.592**	**0.428**
Im_{OLS}	0.806	0.624	0.652	0.459	0.353
K_{OLS}	0.776	0.599	0.620	0.439	0.289
G_{LAD}	0.748	0.612	0.637	0.473	0.363
Im_{LAD}	0.747	0.622	0.646	0.482	0.362
K_{LAD}	0.748	0.542	0.571	0.394	0.278

III: x_{3t} DRAWN FROM A $\chi^2_{(1)}$

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.919**	**0.834**	**0.848**	**0.712**	**0.530**
Im_{OLS}	0.901	0.747	0.771	0.568	0.443
K_{OLS}	0.874	0.713	0.734	0.543	0.356
G_{LAD}	0.853	0.730	0.756	0.589	0.462
Im_{LAD}	0.853	0.742	0.768	0.607	0.460
K_{LAD}	0.846	0.645	0.682	0.487	0.343

IV: x_{4t} DRAWN FROM A CONTAMINATED NORMAL (10%,100)

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	0.341	0.227	0.216	**0.199**	**0.194**
Im_{OLS}	0.320	0.173	0.167	0.141	0.149
K_{OLS}	**0.377**	**0.231**	**0.222**	0.190	0.172
G_{LAD}	0.482	0.432	0.442	0.394	0.276
Im_{LAD}	0.480	0.434	0.443	0.397	0.276
K_{LAD}	0.543	0.498	0.516	0.449	0.267

TABLE 3 (CONTINUED)

V: x_{5t} DRAWN FROM A t_2

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.770**	**0.686**	**0.699**	**0.562**	**0.407**
Im_{OLS}	0.749	0.585	0.614	0.434	0.336
K_{OLS}	0.717	0.557	0.575	0.408	0.278
G_{LAD}	<u>0.683</u>	0.567	0.593	0.456	0.351
Im_{LAD}	0.682	<u>0.580</u>	<u>0.603</u>	<u>0.467</u>	<u>0.352</u>
K_{LAD}	0.682	0.501	0.535	0.376	0.271

VI: x_{1t}, x_{2t} AND x_{3t} TOGETHER

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.505**	**0.488**	**0.469**	**0.423**	**0.287**
Im_{OLS}	0.459	0.323	0.332	0.242	0.221
K_{OLS}	0.423	0.296	0.297	0.222	0.181
G_{LAD}	0.401	0.273	0.290	0.206	<u>0.169</u>
Im_{LAD}	0.395	<u>0.298</u>	<u>0.311</u>	<u>0.226</u>	0.168
K_{LAD}	<u>0.403</u>	0.229	0.240	0.178	0.158

VII: x_{1t}, x_{2t} AND x_{4t} TOGETHER

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.493**	**0.449**	**0.430**	**0.365**	**0.234**
Im_{OLS}	0.448	0.287	0.294	0.203	0.172
K_{OLS}	0.383	0.251	0.257	0.101	0.148
G_{LAD}	<u>0.442</u>	0.304	0.318	0.216	<u>0.182</u>
Im_{LAD}	0.435	<u>0.326</u>	<u>0.338</u>	<u>0.231</u>	0.181
K_{LAD}	0.401	0.244	0.261	0.187	0.167

VIII: x_{1t}, x_{2t}, x_{3t} AND x_{4t} TOGETHER

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.373**	**0.387**	**0.357**	**0.345**	**0.247**
Im_{OLS}	0.334	0.242	0.243	0.202	0.192
K_{OLS}	0.313	0.243	0.239	0.198	0.166
G_{LAD}	0.312	0.234	0.240	0.197	0.177
Im_{LAD}	0.305	<u>0.253</u>	<u>0.255</u>	<u>0.210</u>	0.177
K_{LAD}	<u>0.331</u>	0.232	0.239	0.195	<u>0.179</u>

IX: x_{1t}, x_{2t}, x_{3t} AND x_{5t} TOGETHER

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%,100)$
G_{OLS}	**0.366**	**0.402**	**0.376**	**0.372**	**0.279**
Im_{OLS}	0.322	0.262	0.258	0.219	0.215
K_{OLS}	0.318	0.257	0.259	0.215	0.184
G_{LAD}	0.263	0.218	0.216	0.180	0.168
Im_{LAD}	0.257	<u>0.238</u>	0.236	<u>0.196</u>	0.166
K_{LAD}	<u>0.310</u>	0.215	0.219	0.178	<u>0.172</u>

X: $x_{1t}, x_{2t}, x_{3t}, x_{4t}$, AND x_{5t} TOGETHER

	$N(0,1)$	$\chi^2_{(2)}$	t_5	$\Lambda(0,1)$	$CN(10\%, 100)$
G_{OLS}	**0.279**	**0.332**	**0.302**	**0.314**	**0.237**
Im_{OLS}	0.239	0.207	0.206	0.186	0.182
K_{OLS}	0.250	0.225	0.219	0.190	0.168
G_{LAD}	0.222	0.187	0.194	0.171	0.169
Im_{LAD}	0.214	0.206	0.211	0.185	0.169
K_{LAD}	0.275	0.209	0.219	0.188	0.178

Note: The highest rates of the OLS based tests are in bold, those of the LAD based tests are underlined.

In terms of power we can see the following.

(i) When using OLS residuals, the Glejser test has the highest rejection rates, with few exceptions in part IV. This result is counterbalanced by the poor performance of the Glejser test in terms of the size.

(ii) When using LAD residuals, the Im test presents the highest rates of rejections in most of the experiments.

(iii) The number of rejections reached by the OLS based tests is generally higher than the number of rejections of the same tests computed using LAD residuals. The reduction in power seems to be the pay off of the good size of the LAD based tests.

(iv) Part IV, characterized by contaminated x's, presents the lowest rates of rejections of the first part of the table, reporting the model with only one explanatory variable. In this set of experiments, the LAD based tests have higher rejection rates than their OLS analogues, and the LAD based Koenker test is preferable.

(v) In the experiments with three or more explanatory variables, all the tests have low power.

8 Conclusions

Godfrey (1996) proves that the Glejser test is non-robust in the presence of skewed errors. This paper adds that the Glejser test shows an estimation effect in the presence of error contamination as well. The choice of a robust estimator for the coefficients of the main equation can take care of the estimation effect due to error contamination. By choosing a robust estimator, and by using robust residuals to implement the Glejser test, we can control the impact of contamination on the coefficient estimates and consequently, the impact of contamination on the test function. In addition, we use the directional derivative to define the Taylor expansion of the Glejser test, and this coincides with the normal equation defining the LAD estimator. Thus LAD enters the asymptotic approximation of the Glejser test, and this explains why LAD is a "natural choice" for the implementation of the Glejser test in the case of skewness. Then we discuss the asymptotic distribution of the LAD based Glejser test.

Our simulations show the validity of the LAD based Glejser test in empirical applications. On the basis of the results, we can state that the usual Glejser test based on OLS residuals over-rejects the null not only with skewed, but also with contaminated normal and student-t distributions, that is with heavy tailed errors. The latter cannot be properly analysed by the skewness corrected Glejser tests, since heavy tails and contamination do not necessarily cause asymmetry. Indeed, even the skewness corrected Glejser test over-rejects the true null in the experiments with error contamination, particularly in models having more than one explanatory variable.

However, when we build the usual Glejser test on LAD residuals, the number of rejections is closer to the nominal size. The same can be said for the recently proposed skewness adjusted Glejser test: the performance of the Im test improves when we compute this test using LAD residuals. Furthermore, the improvement achieved by using the LAD based tests is comparable to those achieved by bootstrap, in terms of rejection rates, but LAD has a very low computational burden.

Unfortunately, LAD is not designed to deal with outliers in the explanatory variables. Indeed, the Glejser test based on LAD residuals shows over-rejections in those experiments with contaminated x's. However, this problem cannot be solved by the skewness corrected Glejser test, which causes over-rejection in the same experiments, when it is computed using either OLS or LAD residuals. The search for a different estimator is left to further research.

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