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Point Process Theory and Applications:
Marked Point and Piecewise Deterministic Processes
Martin Jacobsen
Probability and its Applications
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Piecewise deterministic Markov processes (PDMP's) constitute a versatile class of non-diffusion stochastic models. PDMP's, in turn, form a subclass of the so called Marked point processes (MPP's). Markov chains, various queueing models, classical risk processes in insurance, branching processes, processes derived from homogeneous Poisson measures, (which are all PDMP's), renewal processes, semi-Markov processes and likelihood processes are just a few examples coming under the umbrella of MPP's.

The book under review considers MPP's developing randomly over time, in the sense that specific points are associated with times at which certain random events occur. An assumption made throughout is that only finitely many events are allowed in finite intervals of time. The first part of the book deals with the theory, while the second part discusses some applications.

Definition and construction of simple and marked point processes are given in Chapters 2 and 3. Equivalent descriptions, respectively, in terms of counting processes and random counting measures are also provided. Chapter 4 contains the basic theory of probability measures on the space of counting process paths and on the space of discrete counting measures. These distributions on canonical spaces are characterized in terms of compensators, and are described by the structure of certain martingales. The necessary tools of stochastic analysis (*viz.* elementary stochastic integrals and Ito's formula) are developed along the way. Likelihood processes are discussed in Chapter 5, while Chapter 6 deals with stochastic independence between finitely many MPP's, as well as MPP's with independent increments.

Chapter 7 gives an extensive account of PDMP's based on the theory of MPP's. This chapter, together with Chapter 4, can be considered the core of the book. In addition to the pioneering results of Mark Davis, this approach encompasses also nonhomogeneous PDMP's. After outlining the construction, basic properties and a variety of examples, the strong Markov property is established. Various versions of Ito's formula for PDMP's and their implications are presented. Infinitesimal generators, which are usually integro-differential operators, and invariant measures of time homogeneous PDMP's are also discussed.

Chapters 8 to 11 form the second part dealing with applications. One application is a model for soccer leagues in terms of likelihood processes. This model takes into account home-team advantage. This may be timely in this year of FIFA World Cup!

Another application concerns the ruin problem in insurance. For the classical Cramer-Lundberg risk process, ruin probability as well as the joint distribution of ruin time and deficit at ruin (given that ruin time is finite) are derived. The key to the approach is the martingale machinery (developed in Chapter 7) in terms of the space-time generator of the stopped Cramer-Lundberg risk process.

Some other applications are: a simple specific PDMP model to describe price of a finite number of risky assets; GI/G/1 queue and open Jackson queueing network formulated as PDMP's.

The book is well written. Many stochastic processes encountered in "Applied Probability" are exhibited as motivating examples for the theory of MPP's. As the canonical versions of the processes are emphasized, the proofs are more transparent. Though the notations may look bewildering at first sight, they are carefully chosen. This reviewer wishes there had been more exercises. Anyone taking the trouble to work through the details will find the experience rewarding, especially a student of applied probability wanting to have a handle on the techniques of stochastic analysis.

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