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On Testing Exponentiality Against UBA Class of Life Distributions Based On Laplace Transform

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Abstract

The problem of testing various classes of life distributions have been considered in the literature during the last decades. In this paper, we consider a new test statistic for testing exponentiality against used better than age (UBA) class of life distributions based on Laplace transform. This proposed test is presented for complete and right censored data. Furthermore, Pitman asymptotic efficiencies are calculated to assess the performance of our test. Selected critical values are tabulated. Some numerical simulations of power estimates are calculated. Proposed test is presented also in multivariate form. Finally, the test is applied to some real data.

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1 Introduction

Concepts of aging describe how a population of units or systems improves or deteriorates with age. During the past decades, various classes of life distributions have been proposed in order to model different aspects of aging. The best known of these classes are IFR, IFRA, DMRL, NBU, NBUE, and HNBUE. Properties and applications of these aging notions can be found, in Bryson and Siddiqui (1969), Barlow and Proschan (1981), Rolski (1975), Klefsj (1982), and Stoyan (1983). Alzaid (1994) was introduced two class of life distributions who called them the used better than aged (UBA) and the used better than aged in expectation (UBAE). In this paper, we focus on the used better than aged (UBA) class of life distributions.

Let X be a random variable describing the life time of a brand new device which begins to work at time $t=0$. In the reliability literature, X_t denoted the life time of the device of age t with t $\geq$ 0. The probability that the device of age t still working till time x (the survival function) is, $\bar{F}_t(x) = P\left[X > x + t | x > t\right] = \bar{F}(x+t)/\bar{F}(t)$ where $\bar{F}(x)$ the survival function of X.

DEFINITION 1.1. *The distribution function F is said to be used better than age (UBA) if for all x, $t \geq 0$;*

$$\bar{F}_t(x) \geq e^{-\gamma x} \text{ or } \bar{F}(x+t) \geq \bar{F}(t)e^{-\gamma x}.$$

Where, γ is called the asymptotic decay coefficient of X.

DEFINITION 1.2. *The distribution function F is said to be used better than aged in convex ordering (UBAC) if for all x, $t \geq 0$, $\int_x^\infty \bar{F}(u+t)du \geq \bar{F}(t)\int_x^\infty e^{-\gamma u}du$.*

Alzaid (1994) showed that the UBA class is a subclass of UBAE and that if F is IHR (increasing hazard rate), then F is UBA. Similar implications between UBAE, NBUE (new is better than used in expectation) and HNBUE (harmonic new is better than used in expectation) were given by DiCrescenzo (1999). More recently, Willmot and Cai (2000) showed that the UBA class includes the DMRL class (decreasing mean residual lifetime). We thus have

$$\text{IHR} \subset \text{DMRL} \subset \text{UBA} \subset \text{UBAE}$$

For definitions and details of the classes IHR, NBUE and DMRL, see Barlow and Proschan (1981) while for the HNBUE, see Klefsj (1983). Finally, DVRL see Launer(1984). Many papers proposed tests for testing exponentiality against some classes of life distributions based on goodness of fit approach. In this paper, we extend the goodness of fit approach. This paper is organized as follows. In Section 2, we present a test statistic based on Laplace transform for testing H_0: F is exponential against H_1: F is UBA and not exponential. Monte Carlo null distribution critical points are presented for sample size $n = 2(2)50$. In Section 3, Pitman asymptotic efficiency (PAE) of the test for several common distributions is evaluated to assess the performance of our test. In Section 4, the power estimates for sample size $n = 10, 20, 30$ is also calculated. In Section 5, a proposed test is presented for right censored data. In Section 6, a proposed test is presented in multivariate case. Finally, applications using real data are also presented in Section 7.

2 Testing Exponentiality Versus UBA Class Based on Laplace Transform

In this section, we present a test statistic based on Laplace transform for testing H_0: F is exponential versus H_1: F belongs to the class UBA and F is not exponential. We proposed the following measure of departure

$$\delta = \int_0^\infty \int_0^\infty e^{-st}[\bar{F}(x+t) - \bar{F}(t)e^{-\gamma x}]dxdt. \tag{2.1}$$

Wheres is the Laplace transform parameter. We need to prove the following theorem.

THEOREM 2.1. *Let X be the UBA random variable with distribution function F; then based on Laplace Transform technique,*

$$\delta = \frac{1}{s}\left[\mu - \frac{\gamma+s}{\gamma s}E(1-e^{-sX})\right] \tag{2.2}$$

Where μ is the mean life of random variable X.

PROOF. Since,

$$\begin{aligned}\delta &= \int_0^\infty \int_0^\infty e^{-st}\bar{F}(x+t)dxdt - \int_0^\infty \int_0^\infty e^{-st-\gamma x}\bar{F}(t)dxdt \qquad (2.3)\\ &= I_1 - I_2\end{aligned}$$

Where,

$$\begin{aligned}I_1 &= \int_0^\infty \int_0^\infty e^{-st}\bar{F}(x+t)dxdt\\ &= \int_0^\infty \int_t^\infty e^{-st}\bar{F}(w)dwdt\\ &= \frac{\mu}{s} - \frac{1}{s^2}E[1-e^{-st}] \qquad (2.4)\end{aligned}$$

and,

$$\begin{aligned}I_2 &= \int_0^\infty \int_0^\infty e^{-st-\gamma x}\bar{F}(t)dxdt\\ &= \int_0^\infty e^{-st}\bar{F}(t)dt \int_0^\infty e^{-\gamma x}dx\\ &= \frac{1}{\gamma s}E(1-e^{-st}) \qquad (2.5)\end{aligned}$$

From equations, (2.4) and (2.5), we obtain (2.2).

Let $X_1, X_2, ..., X_n$ be a random sample from the distribution function F. For generality, we assume γ is known and equal one. The empirical estimator $\hat{\delta}_n$ of our test statistic can be obtained as

$$\hat{\delta}_n = \frac{1}{ns}\sum_{i=1}^{n}\left[X_i - \frac{1+s}{s}(1-e^{-sX_i})\right],$$

Let us rewrite $\hat{\delta}_n$ as follows

$$\hat{\delta}_n = \frac{1}{ns} \sum_{i=1}^{n} \phi(X_i)$$

where $\phi(X_i) = \frac{X_i}{s} - \frac{1+s}{s^2}(1 - e^{-sX_i})$

To find the limiting distribution of $\hat{\delta}_n$:

Set,

$$\phi(X_1) = \frac{X_1}{s} - \frac{1+s}{s^2}(1 - e^{-sX_1})$$

Then, $\hat{\delta}_n$ is equivalent to U-statistic given by:

$$U_n = \frac{1}{\binom{n}{1}} \sum_i \phi(X_i)$$

The following theorem summarizes the asymptotic normality of $\hat{\delta}_n$.

THEOREM 2.2.

1. *As $n \to \infty$, $\sqrt{n}(\hat{\delta}_n - \delta)$ is asymptotically normal with mean 0 and variance σ^2 where,*

$$\sigma^2 = Var[\hat{\delta}_n] = E\left(\frac{1}{s}\left[X - \frac{1+s}{s}\left(1 - e^{-sX}\right)\right]\right)^2.$$

2. *Under H_0, the variance σ_0^2 is*

$$\sigma_0^2 = \frac{2}{s^2} - \frac{2(1+s)}{s^3} - \frac{2(1+s)}{s^3(1+s)} + \frac{(1+s)^2}{s^4} - \frac{2(1+s)^2}{s^4(1+s)} + \frac{(1+s)^2}{s^4(1+2s)}$$

PROOF.

1. Using standard U-statistic theory, Lee (1990), and direct calculations, we get

$$E[\hat{\delta}_n] = E\left(\frac{1}{s}\left[X - \frac{1+s}{s}\left(1 - e^{-sX}\right)\right]\right);$$

$$\sigma^2 = Var[\hat{\delta}_n] = E\left(\frac{1}{s}\left[X - \frac{1+s}{s}\left(1 - e^{-sX}\right)\right]\right)^2.$$

2. Under H_0, the parameter s must be equal 2, and

$$\mu_0 = E(\hat{\delta}_n) = 0;$$

$$\sigma_0^2 = E[\hat{\delta}_n]^2 = E\left(\frac{1}{s}\left[X - \frac{1+s}{s}\left(1-e^{-sX}\right)\right]\right)^2$$

$$= \frac{2}{s^2} - \frac{2(1+s)}{s^3} - \frac{2}{s^3} + \frac{(1+s)^2}{s^4} - \frac{2(1+s)}{s^4} + \frac{(1+s)^2}{s^4(1+2s)}.$$

$$= \frac{11}{80}.$$

Table 1: Critical value for $\hat{\delta}_n$

n	95 %	98 %	99 %
2	0.9627	1.0595	1.1249
4	0.9628	1.0313	1.0775
6	0.9458	1.0017	1.0395
8	0.9311	0.9795	1.0123
10	0.9189	0.9623	0.9915
12	0.9098	0.9493	0.9760
14	0.9009	0.9375	0.9623
16	0.8937	0.9279	0.9511
18	0.8876	0.9199	0.9417
20	0.8822	0.9129	0.9336
22	0.8773	0.9065	0.9263
24	0.8731	0.9010	0.9199
26	0.8690	0.8959	0.9140
28	0.8656	0.8914	0.9089
30	0.8626	0.8876	0.9045
32	0.8594	0.8836	0.8999
34	0.8566	0.8801	0.8960
36	0.8541	0.8770	0.8929
38	0.8520	0.8742	0.8892
40	0.8496	0.8713	0.8859
42	0.8476	0.8688	0.8830
44	0.8456	0.8662	0.8802
46	0.8438	0.8640	0.8776
48	0.8422	0.8619	0.8753
50	0.8406	0.8600	0.8731

To conduct the test, calculate $\sqrt{n}\hat{\delta}_n/\sigma_0$ and reject H_0 if this value exceeds the standard normal value $Z_{1-\alpha}$. To illustrate the test, we have simulated the upper percentile points for the significance level $\alpha = 0.01$, 0.02, and 0.05. The calculation of the test $\hat{\delta}_n$ is based on 10,000 simulated samples from the standard exponential distribution. Table 1 gives the critical values of the test statistic $\hat{\delta}_n$. Figure 1 shows the critical values of the test statistic $\hat{\delta}_n$ are decreasing as the sample size increasing as follow:

3 The Pitman Asymptotic Efficiency (PAE)

For the test suggested above, the *PAE* is computed to assess the performance of our test, where

$$PAE(\delta) = \frac{\left|\frac{d}{d\theta}\delta_\theta\right|_{\theta\to\theta_0}}{\sigma_0}$$

Where θ is unknown parameter in such a way that $\theta = \theta_0$ yields a distribution belonging to the null hypothesis (exponential distribution) whereas $\theta > \theta_0$ yields distributions from alternative (Linear failure rate, Makeham and Weibull).

At, s = 2, $\gamma = 1$

$$\delta_\theta = \frac{\mu_\theta}{2} + \frac{3}{2}E_\theta(e^{-2X}) - \frac{3}{4};$$

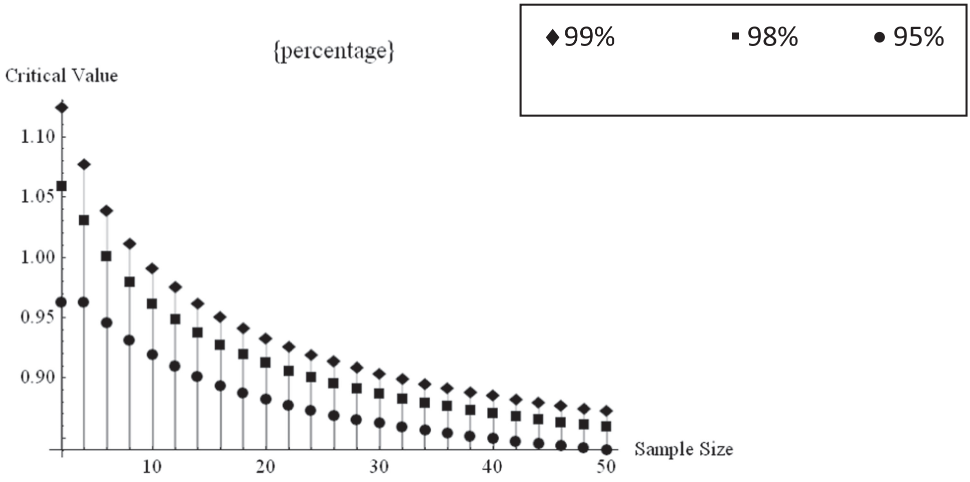

Figure 1: The relation between sample size and critical values

where

$$\mu_\theta = \int_0^\infty \bar{F}_\theta(x)dx, E_\theta(e^{-2X}) = \int_0^\infty e^{-2x}dF_\theta(x), \text{ and } \sigma_0^2 = \lim_{n\to\infty}[Var(\delta_\theta)]_{\theta\to\theta_0}.$$

Hence, $PAE[\delta] = \left[\frac{\mu'_\theta}{2} + \frac{3}{2}\frac{d}{d\theta}E_\theta\right]_{\theta\to\theta_0}$

Where, $\mu'_\theta = \int_0^\infty \bar{F}'_\theta(x)dx$.

Now, we evaluate the PAE for some commonly used distribution in reliability. These are

1. Linear failure rate: $\bar{F}_\theta(x) = e^{-\left(x+\frac{1}{2}\theta x^2\right)}$, $x > 0$, $\theta \geq 0$;
2. Makeham: $\bar{F}_\theta(x) = e^{-x-\theta(x-1+e^{-x})}$, $x > 0$, $\theta \geq 0$;
3. Weibull: $\bar{F}_\theta(x) = e^{-x^\theta} x > 0,\ \theta > 0$.

Under H_0, we evaluate PAE for the above distributions; we get the result in Table 2 as follows

Where, $\sigma_0 = \sqrt{\frac{11}{80}}$, the efficiency of our test is increasing in s = 2. So our test has a good efficiency if it compares with other tests.

4 Power Estimates

The power estimate of the test statistic $\hat{\delta}_n$ is useful in clarifying how much the test can detect the departure from exponentiality towards the class UBA. The higher value of the power estimate indicates that the test statistic is more able to detect such a departure. The power of the test statistics $\hat{\delta}_n$ is considered for 5 % percentile in Table 3 for three alternatives. These alternatives are:

1. Linear failure rate: $\bar{F}_\theta(x) = e^{-\left(x+\frac{1}{2}\theta x^2\right)}$, $x > 0$, $\theta \geq 0$;
2. Makeham: $\bar{F}_\theta(x) = e^{-x-\theta(x-1+e^{-x})}$, $x > 0$, $\theta \geq 0$;
3. Weibull: $\bar{F}_\theta(x) = e^{-x^\theta}$, $x > 0,\ \theta > 0$.

Table 2: PAE of $\hat{\delta}_n$

Distribution	PAE
Linear failure rate	$\frac{1}{\sigma_0}\left[\frac{1}{s} - \frac{s+1}{s(1+s)^3}\right]$
Makeham	$\frac{1}{\sigma_0}\left[\frac{1}{s} - \frac{s+1}{s(1+s)^2(2+s)}\right]$
Weibull	$\frac{(s+1)[EulerGamma+Log(s)]}{es\sigma_0}$

For appropriate values of θ, these distributions can be reduced to the exponential distribution. The power estimate of the test statistic $\hat{\delta}_n$, given in Table 3 shows the chance of detecting departure from exponentiality towards the UBA property as θ increases, or the sample size n increases for the linear failure rate, Makeham, and Weibull distribution.

5 Test for UBA in Case for Right Censored Data Based on Laplace Transform

In this section, a test statistic proposed to test H_0 versus H_1 with randomly right censored samples. In the censored model, instead of dealing with $X_1, X_2, ..., X_n$, we observe the pair (Z_i, ξ_i), i=1,2,..., n, where $Z_i = \min(X_i, Y_i)$ and $\xi_i = 1$if $Z_i = X_i$, $\xi_i = 0$ if $Z_i = Y_i$, where $X_1, X_2, ..., X_n$ denote their true life time from a distribution F and $Y_1, Y_2, ..., Y_n$ be i.i.d. according to distribution G. Also X's and Y's are independent. For generality, we assume γ is known and equal one. Let $Z_{(0)} = 0 \leq Z_{(1)} \leq Z_{(2)} \leq ... \leq Z_{(n)}$ denote the order Z's and $\xi_{(i)}$ is the δ_i corresponding to $Z_{(i)}$, respectively. Using the Kaplan Meier estimator in the case of censored data (Z_i, ξ_i), i=1...n, the proposed test statistic for right censored data is given by

$$\hat{\delta}_n^c = \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} C_k^{\xi(k)} \right) \left(Z_{(j)} - Z_{(j-1)} \right) - \frac{1+s}{s} e^{-sZ_{(j)}} \left(\prod_{k=1}^{i-2} C_k^{\xi(k)} - \prod_{k=1}^{i-1} C_k^{\xi(k)} \right) \tag{5.1}$$

Where:

$$\hat{\mu} = \sum_{j=1}^{l} \prod_{k=1}^{j-1} C_k^{\xi_k} (Z_{(j)} - Z_{(j-1)})$$

Table 3: Power Estimates for $\hat{\delta}_n$

Distribution	θ	n = 10	n = 20	n = 30
Linear failure rate	1	0.8970	0.9933	0.9996
	2	0.9674	0.9995	0.9999
	3	0.9801	0.9998	0.9999
Makeham	1	0.7675	0.9569	0.9933
	2	0.9320	0.9974	0.9999
	3	0.9658	0.9994	0.9999
Weibull	2	0.8810	0.9906	0.9994
	3	0.9584	0.9991	0.9999
	4	1	1	1

$$d\hat{F}_n(Z_j) = \left[\prod_{q=1}^{j-2} C_q^{\xi_q} - \prod_{q=1}^{j-1} C_q^{\xi_q}\right]$$

and

$$\hat{\hat{F}}_n(t) = \prod_{m<Z_{(m)}<t} C_m^{\xi_m} \text{ Where } C_m = \frac{n-m}{n-m+1} \text{ and } t \in [0, Z_{(m)}].$$

To illustrate the test, we have simulated the upper percentile points for the significance level $\alpha = 0.01$, 0.02, and 0.05. The calculation of the test $\hat{\delta}_n^C$ is based on 10,000 simulated samples from the standard exponential distribution. Table 4, gives the critical values of the test statistic $\hat{\delta}_n^C$. Figure 2

Table 4: Critical value of $\hat{\delta}_n^C$

n	95 %	98 %	99 %
2	2.5672	2.7299	2.8399
4	0.6096	0.7247	0.8025
6	0.4977	0.5917	0.6552
8	0.4310	0.5124	0.5674
10	0.3855	0.4583	0.5075
12	0.3159	0.4184	0.4633
14	0.3258	0.3873	0.4289
16	0.3048	0.3623	0.4012
18	0.2873	0.3416	0.3783
20	0.2726	0.3241	0.3588
22	0.2599	0.3090	0.3421
24	0.2488	0.2958	0.3276
26	0.2391	0.2842	0.3147
28	0.2304	0.2739	0.3033
30	0.2226	0.2646	0.2930
32	0.2155	0.2562	0.2837
34	0.2910	0.2485	0.2752
36	0.2032	0.2415	0.2675
38	0.1977	0.2351	0.2603
40	0.1927	0.2291	0.2537
42	0.1881	0.2236	0.2476
44	0.1838	0.2185	0.2419
46	0.1797	0.2137	0.2366
48	−1.4961	−1.4629	−1.9405
50	−0.6451	−0.6126	−0.5906

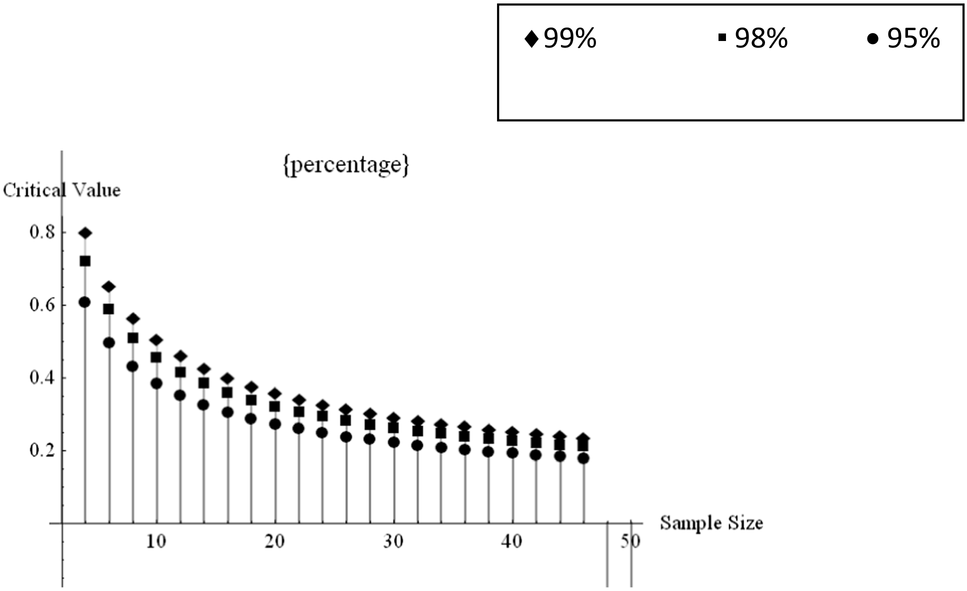

Figure 2: The relation between sample size and critical values (censored data)

shows the critical values of the test statistic $\hat{\delta}_n^C$ are decreasing as the sample size increasing as follow:

6 Test for UBA Based on Laplace Transform in Multivariate Case

A complex system usually consists of several components which are working under the same environment and hence their lifetimes are, generally, dependent. In the literature several attempts have been made to extend the concepts of aging to the multivariate case. Some important references are Brindley and Thompson (1972), Arjas (1981), Savits (1985), Shaked and Shanthikumar (1988, 1991), Barlow and Mendel (1993), and Barlow and Spizzichino (1993, 1999), among others. In this paper, we present the definition of multivariate used better than age (MUBA).

Definition 6.1. *The distribution function $F(\underline{x})$is said to be multivariate used better than age (MUBA) if for all $\underline{x} = (x_1, ..., x_k)$, $\underline{t} = (t_1, ..., t_k)$? 0;*

$$\bar{F}(\underline{x} + \underline{t}) \geq \bar{F}(\underline{t}) e^{-\gamma \sum_{i=1}^{k} x_i}.$$

Where, γ is called the asymptotic decay coefficient of $\underline{x} = (x_1, ..., x_k)$.

The equality in the above definition holds if and only if $X_1, ..., X_k$ are independent exponential random variables. Now, we present a test statistic for testing H_0: $\bar{F}(x_1, .., x_k)$ is multivariate exponential (MVE) with independent marginals i.e. $\bar{F}(x_1, ..., x_k) = e^{-\lambda(x_1+...+x_k)}$ versus H_1: $\bar{F}(x_1, .., x_k)$ is MUBA and not exponential. We proposed the following measure of departure

$$\begin{aligned}\delta_{MUBA} &= \int_0^\infty \int_0^\infty e^{-s(t_1+...+t_K)}[\bar{F}(x_1+t_1, ..., x_k+t_k) \\ &\qquad -e^{-\gamma(x_1+...+x_k)}\bar{F}(t_1, .., t_k)]d\underline{x}d\underline{t}\end{aligned}$$

THEOREM 6.1. *Let $\underline{x}$ be a MUBA with survival function $\bar{F}(x_1, ..., x_k) = e^{-\lambda(x_1+...+x_k)}$, then, based on Laplace transform techniques*

$$\delta_{MUBA} = \frac{1}{s^k}\left[E(X_1, ..., X_k) - \prod_{i=1}^{k} \frac{(\gamma+s)}{\gamma s} E(1-e^{-sX_i})\right] \tag{6.1}$$

PROOF. Note that

$$\begin{aligned}\delta_{MUBA} &= \int_0^\infty \int_0^\infty e^{-s(t_1+...+t_K)}\left[\bar{F}(x_1+t_1, ..., x_k+t_k)\right. \\ &\qquad \left. -e^{-\gamma(x_1+...+x_k)}\bar{F}(t_1, .., t_k)\right]d\underline{x}d\underline{t} \\ &= I_1 - I_2\end{aligned}$$

Where, $I_1 = \int_0^\infty \int_0^\infty e^{-s(t_1+...+t_k)}\bar{F}(x_1+t_1, ..., x_k+t_k)d\underline{x}d\underline{t}$

$$= \frac{1}{s^k}\left[E(X_1, ..., X_k) - \prod_{i=1}^{k} \frac{1}{s^k} E(1-e^{-sX_i})\right] \tag{6.2}$$

and
$$I_2 = \int_0^\infty \int_0^\infty e^{-s(t_1+...+t_k)-\gamma(x_1+...+x_k)}\bar{F}(t_1, ..., t_k)d\underline{x}d\underline{t}$$

$$= \frac{1}{\gamma s^k}\prod_{i=1}^{k} E(1-e^{-sX_i}) \tag{6.3}$$

From, (6.2), (6.3) we get (6.1).

By using a random samples $X_{1i}, ...X_{ki}$ of size n. For generality, we assume γ is known and equal one. The empirical estimate of $\hat{\delta}_{MUBA}$ of δ_{MUBA} can obtained by

$$\hat{\delta}_{MUBA} = \frac{1}{ns^k}\left[\left(\sum_{i=1}^{n}(X_{1i}, ..., X_{ki}) - \frac{1}{n}\sum_{i=1}^{n}\prod_{j=1}^{k}\left(\frac{1+s}{s}\right)^k (n - e^{-sX_{ji}})\right]$$

$$= \frac{1}{s^k}\left[\theta_1 - \theta_2^k\right],$$

where, $\theta_1 = \frac{1}{n}\sum_{i=1}^{n}(X_{1i}, ..., X_{ki})$ and $\theta_2^k = \frac{1}{n}\sum_{i=1}^{n}\prod_{j=1}^{k}\left(\frac{1+s}{s}\right)^k (n - e^{-sX_{ji}})$.

The following theorem summarizes the asymptotic normality of $\hat{\delta}_{MUBA}$.

THEOREM 6.2. *As $n \to \infty$, $\sqrt{n}(\hat{\delta}_{MUBA} - \delta_{MUBA})$ is asymptotically normal with mean 0 and variance*

$$\sigma^2 = Var[\hat{\delta}_{MUBA}] = \frac{1}{s^k}\left[\sigma_{11} + k^2\theta_2^{2(k-1)}\sigma_{22} - 2k\theta^{k-1}\sigma_{12}\right]$$

PROOF. Note that,

$$\hat{\delta}_{MUBA} = \frac{1}{s^k}[\theta_1 - \theta_2^k]$$

Where, $\sigma_{11} = s^k nVar(\theta_1), \sigma_{22} = s^k nVar(\theta_2)$, $\sigma_{12} = s^k nCov(\theta_1, \theta_2)$.

Now asymptotic variance of $\hat{\delta}_{MUBA}$ is

$$\sigma^2 = Var[\hat{\delta}_{MUBA}] = \frac{1}{s^k}\left(\left[\frac{\partial\hat{\delta}_{MUBA}}{\partial\theta_1}\right]^2\sigma_{11} + \left[\frac{\partial\hat{\delta}_{MUBA}}{\partial\theta_2}\right]^2\sigma_{22} - 2\left[\frac{\partial\hat{\delta}_{MUBA}}{\partial\theta_1}\right]\left[\frac{\partial\hat{\delta}_{MUBA}}{\partial\theta_2}\right]\sigma_{12}\right)$$

$$\sigma^2 = Var[\hat{\delta}_{MUBA}] = \frac{1}{s^k}\left[\sigma_{11} + k^2\theta_2^{2(k-1)}\sigma_{22} - 2k\theta^{k-1}\sigma_{12}\right]$$

Under H_0, $E(\hat{\delta}_{MUBA}) = 0$ and the variance of $\hat{\delta}_{MUBA}$ is a function of $\lambda_1, ..., \lambda_k$.

7 Application

7.1. Application for Complete Data.

EXAMPLE 1. The following data represent 39 liver cancers patients taken from El Minia Cancer Center Ministry of Health Egypt Attia et al. (2004) the ordered life times (in days) are:

10; 14; 14; 14; 14; 14; 15; 17; 18; 20; 20; 20; 20; 20; 23; 23; 24; 26; 30; 30;

31; 40; 49; 51; 52; 60; 61; 67; 71; 74; 75; 87; 96; 105; 107; 107; 107; 116; 150:

It was found that the test statistic for the data set, $\hat{\delta}_n = -23.63$, which less than the critical value of Table 1. Then we accept the null hypothesis of exponentiality.

7.2. Application for Censored Data. The following data represent 39 liver cancers patients taken from El Minia Cancer Center Ministry of Health Egypt Attia et al. (2004) the ordered life times (in days) are:

(i) Non-censored data

$$10; 14; 14; 14; 14; 14; 15; 17; 18; 20; 20; 20; 20; 20; 23; 23; 24; 26; 30; 30;$$

$$31; 40; 49; 51; 52; 60; 61; 67; 71; 74; 75; 87; 96; 105; 107; 107; 107; 116; 150.$$

(ii) Censored data

$$30; 30; 30; 30; 30; 60; 150; 150; 150; 150; 150; 185:$$

It was found that the test statistic for the data set, $\hat{\delta}_n^C = 3.6$, which it increases the critical value of Table 4. Then we reject the null hypothesis of exponentiality.

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